Labatorial 1: Density of Granular Solids[1]

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Pre-Reading:

- *Physics for Scientists and Engineers* by Serway and Jewett (10th ed.), Section:
  - 1.4 – Conversion of Units
  - 1.6 – Significant Figures
- The first two sections of serc.carleton.edu/mathyouneed/density/index.html
- Sections 1 and 2 of http://lectureonline.cl.msu.edu/~mmp/labs/error/e2.htm

Equipment: Small beaker, thermometer, triple beam balance, pycnometer, glass beads, strainer, acetone, calculator

Learning Goals:

- Understanding how solids immersed in a liquid displace the liquid
- Understanding how measurement error is propagated through calculations

Pycnometers: A pycnometer is a small flask with a glass stopper. A capillary (i.e. thin) opening, which runs along the length of the stopper, makes it possible to fill the pycnometer completely – that is, without leaving a bubble of air in the flask. (Note: After use, a pycnometer can be dried by rinsing it with some acetone and then dumping the liquid in a wastebasket.)

Activity 1: Some Conceptual Warm-Ups (15-25 min.)

Question 1:

A beaker is filled with $100\ mL$ of water, whose density is $1\ g/mL$. If the water and the beaker together have a mass of $0.250\ kg$, what is the mass of the beaker?

Question 2:

An irregular chunk of metal has a density of $7800\ kg/m^3$. A $15\ g$ chunk of the metal is added into a graduated cylinder filled with enough liquid to submerge the chunk. If the final volume of the cylinder reads $40.2\ mL$, what was the initial volume? (Recall that $1\ cm^3 = 1\ mL$.)

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[1] The experiment performed in this labatorial has been adapted from the 2018-2019 lab manual for the course "PHYS 224: Introduction to Experimental Mechanics" at Concordia University.

Checkpoint 1: Before moving on to the next part, have your instructor check the results you obtained so far.

Activity 2: Three Blind Mice (25-35 min.)

<u>Problem</u>: A team of scientists wish to measure out a precise quantity of a liquid that they produced in the lab, as well as quantify the uncertainties in their measurement. (The amount itself is important, but they want to be able to use the same volume consistently for future experiments and know how precise their procedure is.) However, they miraculously crash into each other, shattering all of their reading glasses in the process. They can still read the readings from their digital thermometer and triple-beam balance, but the light markings on a graduated cylinder are no longer legible to them. Still determined to take the measurement, however, they decide to use the remaining tools at their disposal: a pycnometer, a digital thermometer, and a triple-beam balance.

Question 3:
Due to various effects including thermal expansion, the density of water changes with temperature. Therefore, we need to know the temperature of the water for the volume measurements to be accurate. (Thermal expansion can also cause the pycnometer itself to deform, affecting its volume, but can be avoided by handling the pycnometer by its neck with one or two layers of paper towel.) Fill the large beaker halfway with water from the sink. Leave the thermometer in the beaker for a few minutes and then take a reading of its temperature. What is the density of the water? Circle your answer and convert it to units of $g\backslash mL$. (While you wait for the thermometer, feel free to begin working on the next question.)

Table 1 – Density of water (in kg/m^3, all with uncertainty $\pm$ 0.02) at various temperatures.

$\boldsymbol{T}$ (°$\boldsymbol{C}$)	18	19	20	21	22	23	24	25	26
.0	998.60	998.40	998.20	998.00	997.80	997.60	997.40	997.20	997.00
.1	998.58	998.38	998.18	997.98	997.78	997.58	997.38	997.18	996.98
.2	998.56	998.36	998.16	997.96	997.76	997.56	997.36	997.16	996.96
.3	998.54	998.34	998.14	997.94	997.74	997.54	997.34	997.14	996.94
.4	998.52	998.32	998.12	997.92	997.72	997.52	997.32	997.12	996.92
.5	998.50	998.30	998.10	997.90	997.70	997.50	997.30	997.10	996.90
.6	998.48	998.28	998.08	997.88	997.68	997.48	997.28	997.08	996.88
.7	998.46	998.26	998.06	997.86	997.66	997.46	997.26	997.06	996.86
.8	998.44	998.24	998.04	997.84	997.64	997.44	997.24	997.04	996.84
.9	998.42	998.22	998.02	997.82	997.62	997.42	997.22	997.02	996.82

Review of Measurements, Uncertainties, and Error Propagation:

The smallest division of scale markings on an instrument is called the **absolute error**. When we want to report a measurement such as the width of a paper, we represent it as: $W \pm \Delta W$, where *W* is the measurement value and *ΔW* is the measurement uncertainty or absolute error. This means that the true width will probably lie somewhere in the interval $(W - \Delta W, W + \Delta W)$. If you take two different width measurements with their own absolute errors and their intervals overlap, then we say that *the two measurements agree within uncertainty*.

If you have measured values for the quantities *X*, *Y*, and *Z* with uncertainties *ΔX*, *ΔY*, and *ΔZ*, when **adding or subtracting** two measurements that have an error, **add the absolute errors** to give you the absolute error in the sum or difference.

$$\text{If} \quad R = X + Y \quad \text{then} \quad \Delta R = \Delta X + \Delta Y$$

$$\text{If} \quad R = X - Y \quad \text{then} \quad \Delta R = \Delta X + \Delta Y \qquad (1)$$

The **relative error** of a measurement is defined as:

$$Relative\ Error = \frac{Absolute\ Error}{Measured\ Value}$$

When **multiplying or dividing** two measurements that have an error, **add the relative errors** to give you the relative error in the product or quotient.

$$\text{If} \quad R = X \cdot Y \quad \text{then} \quad \frac{\Delta R}{R} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y}$$

$$\text{If} \quad R = \frac{X}{Y} \quad \text{then} \quad \frac{\Delta R}{R} = \frac{\Delta X}{X} + \frac{\Delta Y}{Y} \qquad (2)$$

Therefore, the absolute error in this case will be given by

$$\Delta R = R\left(\frac{\Delta X}{X} + \frac{\Delta Y}{Y}\right) \qquad (3)$$

Question 4:
Returning back to the experiment and keeping in mind that we always need to allow air bubbles to rise to the top of a pycnometer after it is filled with liquid, let us devise the procedure the blinded scientists would have to follow in order to measure the volume of the pycnometer, and use it to measure your pycnometer's volume.

a. Measure the mass of the empty pycnometer with its stopper and the pycnometer filled with water (closed with the stopper).

M_p = ____________________ $\pm 0.1\ g$ M_{wp} = ____________________ $\pm 0.1\ g$

b. Calculate the mass of the water in the pycnometer, using Equation 1 to calculate its uncertainty.

M_w = ____________________ $\pm$ ___________ g

c. Calculate the volume of the pycnometer, using Equation 3 to calculate its uncertainty. (Ask the TA for how to quote this uncertainty properly.)

V_p = ____________________ $\pm$ ___________ mL

d. Calculate the percentage difference between your measured value for V_p and the calibrated value of $50\ mL$ at 20 ℃.

e. How does using percentage difference differ from using uncertainty in quantifying measurement error and interpreting the precision of results?

Checkpoint 2: Before moving on to the next part, have your instructor check the results you obtained so far.

Activity 3: Exploring Worlds Unknown (60-70 min.)

Problem: In an ongoing space search by NASA, a potentially habitable planet is discovered by an extra-solar probe. One of the crucial conditions for the habitability of a planet is the suitable composition of its soil (i.e. containing carbon-based compounds similar to the Earth's soil). To verify this, the probe launches a lander to the surface of this rocky planet in order to take some measurements. Luckily, it happens to find some nearby dry soil that it can easily collect. However, the soil, being very brittle, crumbles upon being lifted, and so only the granular form of the solid could be collected. Using standardized procedures, it begins measuring the density of the soil to verify whether not the new planet meets habitability conditions. You will now put yourself in the shoes of those designing the procedures, testing them out first in your lab.

Question 5:

a. Suppose we pour $20\ mL$ of water at $20\ °C$ into a graduated cylinder. We then insert a $1\ cm^3$ cube of lead with density $11.34\ g/cm^3$ into the cylinder. What will be the final volume measured in the graduated cylinder?

b. Now suppose we fill this cylinder to maximum capacity and then insert the same cube, causing water to overflow. Compare this cylinder with another cylinder filled with only water. What is the difference in mass between the two cylinders?

Question 6:

We now want to measure the density of the granular solid (which, for us, is made of glass).

a. Place a small beaker (dry and empty) on the triple beam balance. Record its mass.

   M_b = ______________________ $\pm$ ________________ g

b. Carefully add granular solid to this beaker until there is $50\ g$ of the solid. (If the solids are not dried, place them in the strainer and slowly pour some acetone on them. You can shake the strainer around slightly while they evaporate to speed up the drying process.)

c. Pour out about half of the water from the pycnometer into the sink. Then add the granular solid to the pycnometer, top it off with water, and insert the stopper. What is the total mass of the pycnometer with its content (i.e. water and solid)?

   M_{wsp} = ______________________ $\pm$ ________________ g

d. The difference between the solid's mass M_s and the displaced water's mass m_{disp} can be expressed in terms of our measurements as $M_s - m_{disp} = M_{wsp} - M_{wp}$. Then what is the mass of the displaced water?

m_{disp} = ________________________ $\pm$ _________________ g

e. What is the volume of the granular solid in the pycnometer? (Hint: The water gets displaced by the solid.)

V_s = _________________________ $\pm$ _________________ mL

f. What is the density of the solid?

ρ_s = _________________________ $\pm$ _________________ kg/m^3

g. How does your answer compare to the tabulated value of $2400\ kg/m^3$? Does it agree within uncertainty? Why do you think that is?

h. Given what we have learned about how errors propagate through calculations, how can we improve this experiment to reduce the uncertainties in our results?

Checkpoint 3: Please <u>clean your area</u> and then have your instructor check your work before leaving the lab.

Component	**Explanations**	**Points**	**Mark**
Worksheet	• If you finish all checkpoints, you will get 4 points.	4	
Group	• All students must be engaged in the lab activity. • All students must work, discuss, and share their information in the lab. • Interaction with group members and TA is mandatory. • All students must obtain answers to the questions that are the same as the other group members.	3	
Individual	• All appropriate data must be collected. • Data must be well organized and neatly displayed, including graphs. • The results of calculations must be presented with appropriate units. • Related physics concepts must be stated correctly.	3	

Please note that:

- Not properly cleaning the worktable or not putting away equipment that was taken out will result in a 1-point deduction from the "group" component of all members' grades.
- As of Labatorial 2 onward, not bringing your labatorial manual (in which case, a separately printed worksheet will be provided) or pre-reading summary to the lab will result in a 1-point deduction from the "individual" component of your grade.
- Progressing as a group is critical to the success of the labatorial, and so being more than 15 minutes late will result in a 1-point deduction from the "individual" component of your grade. Being more than 20 minutes late means you cannot perform the labatorial and you will receive a 0.