Labatorial 5: The Coefficient of Restitution and *g*, the Acceleration Due to Gravity[5]

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Pre-Reading:

- *Physics for Scientists and Engineers* by Serway and Jewett (10th ed.), Section:
  - 2.7 – A.M.: Particle Under Constant Acceleration (ignore examples)
  - 2.8 – Freely Falling Objects
  - 9.1 – Linear Momentum
  - 9.4 – Collisions in One Dimension (ignore mathematical parts)

Equipment: Glass tube, meter stick, small steel ball, stopwatch

Learning Goals:

- Understanding how the coefficient of restitution describes how elastic or inelastic a collision is
- Understanding how to determine the acceleration of gravity
- Understanding the connection between the coefficient of restitution and gravity

Activity 1: Michael Jordan's Energy Lawsuit (20-30 min.)

Problem: Michael Jordan, star basketball player, does not perform well one day. He claims that the basketball he was given that day was not bouncy enough, that it should be able to return to his hand after dropping it without applying any force whatsoever. But the referees want him to understand that the bouncing of basketballs is not perfectly elastic, and so kinetic energy ($K = mv^2/2$) is partially converted to other forms.

Question 1:

When an object is dropped and collides with the ground, it may bounce back up, but not necessarily back to up the height at which it started. If it does go all the way back up, it means that it maintained all of its kinetic energy before and after the collision, and we call it an **elastic collision**. If the object does not bounce at all, then its kinetic energy was completely converted to other forms, and we call it a **perfectly inelastic collision**. How is it possible for kinetic energy to not be conserved? What other forms of energy could it convert to? Try giving an intuitive physical explanation.

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[5] The experiment performed in this labatorial has been adapted from: Farkas, N., and R. D. Ramsier. "Measurement of coefficient of restitution made easy." *Physics education* 41.1 (2006): 73.

Question 2:

Consider a basketball that is dropped from rest and bounces. Suppose that some kinetic energy is converted into other forms after impact. However as you know from experience, not all collisions are the same, with some being more inelastic than others. We can define a quantity called the **coefficient of restitution** e, which gives a measure of how inelastic a collision is. The coefficient of restitution for the impact of a basketball as previously described (i.e. object dropped from rest and colliding with an effectively stationary object) is $e = -v/u$, where u and v are the 1-dimensional *vector* velocities of the ball before and after impact, respectively.

a. Given the definition, what is the range of values that the coefficient of restitution can take on? What are the values for an elastic and perfectly inelastic collision?

b. How much kinetic energy is lost (i.e. converted) at impact?

c. What is the relative loss in kinetic energy (i.e. the fraction of converted kinetic energy) at impact in terms of **only** e?

Although you have helped Jordan understand that basketballs do not collide perfectly elastically, he is still convinced that his ball was rigged somehow. The NBA standard for relative kinetic energy loss for basketballs is supposed to be 0.2 or less. So naturally, Jordan's team of on-hand NASA scientists want to check if this true. By comparison, they find that their steel ball bounces on a steel block just like Jordan's basketball does on a gymnasium floor, and so they decide to use this smaller setup (which is the same one you have) for their experiment. However, they need some time to finish a project that will help Jordan with his problem, and so discovering the conclusion to Jordan's lawsuit will have to wait…

Checkpoint 1: Before moving on to the next part, have your instructor check the results you obtained so far.

Activity 2: The Power of a Humble Space Probe 1 (20-30 min.)

Problem: NASA wants to send a lander to an unknown planet and use it to figure out its properties. But in transit, the lander accidentally crash-lands on an enormous, round asteroid that they had missed when planning its trajectory. Unfortunately, besides its communications system, nearly all of its measurement devices get damaged. However, its sonar system was left unscathed, leaving the lander still capable of measuring distances and times. As unfortunate as this crash landing may be, they want to make the most of it by learning whatever they can about the asteroid, which could provide insight into the composition of objects in that region of space.

Question 3:

a. They first decide to determine the strength of the gravitational acceleration on the surface of the asteroid. The lander is capable of dropping various objects (from rest) from different heights and measuring the time they take to reach the ground. It starts by dropping small steels balls, as you are given in the lab. Given this limited functional capability, explain how can the lander figure out the acceleration of gravity on the asteroid using kinematics (i.e. describe the measurements it would need to take and how it would use that data to obtain g)? You should sketch a drawing and graph to help with your explanation. Note that we are not asking you to perform the experiment here.

b. Following this procedure on Earth, does our answer depend on the object used? For example, what if we had used a Styrofoam cup? How about on the asteroid?

Checkpoint 2: Before moving on to the next part, have your instructor check the results you obtained so far.

Activity 3: The Power of a Humble Space Probe 2 (80-90 min.)

Question 4:
Lastly, NASA decides to determine the coefficient of restitution of the asteroid, again using nothing but the distance- and time-measuring capabilities of the lander. We will work through the steps that allow them to do that (and ultimately, decide Jordan's fate). When a partially inelastic object bounces, its height as a function of time looks as follows:

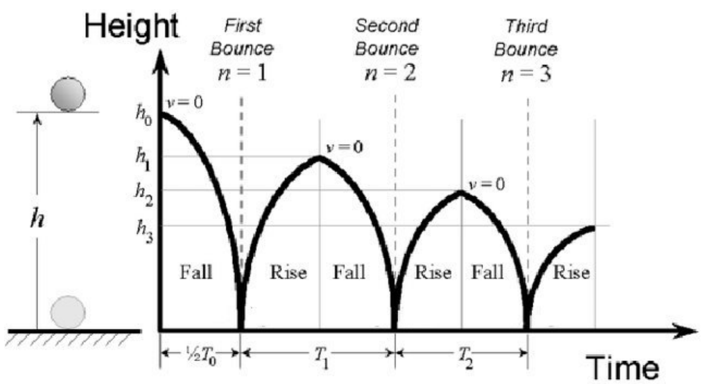

Figure 1 – Height of a bouncing ball as a function of time.

a. How much time is required for the initial descent?

b. Now, it can be shown from conservation of energy that the coefficient of restitution can be expressed as $e = \sqrt{h_{i+1}/h_i}$, i.e. the square root of the ratio of successive bounce heights. How much time elapses between the 1st and 2nd bounce? How about between the 2nd and 3rd and so on? Express your answers using only e, h_0, and g. (Hint: How can we use the definition of e to relate any h_i back to h_0?) Feel free to ask the TA for help with the algebra steps here and in part (c).

c. Although we do see a ball stop bouncing eventually after dropping it, we can also imagine that it keeps bouncing indefinitely, but with each bounce getting smaller and smaller to the point that we can no longer see it moving. By adding together the times calculated in the above two questions, what is the total time required for the ball to stop moving? Express your answer as an infinite sum. (Formal summation notation is not necessary; just using '…' to represent the rest of the terms is fine.) Then using the fact that $1 + x + x^2 + x^3 + \cdots = 1/(1 - x)$ for $|x| < 1$, simplify this expression.

Checkpoint 3: Before moving on to the next part, have your instructor check the results you obtained so far.

d. This result tells us that the total time t_{tot} for the bouncing ball to stop is proportional to the square root of the drop height $\sqrt{h_0}$. Therefore, all the other coefficients will together be equal to the slope of a t_{tot} vs. $\sqrt{h_0}$ graph, which you should group together and call a. Being that NASA's lander can measure distances and times, it is then capable of determining the coefficient of restitution of the asteroid. Describe the experimental steps it needs to take, and perform the experiment with your small steel ball in the lab, using the graph paper at the end of the worksheet for your graph.

e. Draw a line of best fit for the t_{tot} vs. $\sqrt{h_0}$ graph and calculate its slope.

f. Along with any other constants, substitute your slope value from Question 4e into the slope equation from Question 4d and solve for the coefficient of restitution of the asteroid (and thus Jordan's basketball).

g. Was Jordan's ball up to NBA standards? That is, is the relative kinetic energy loss less than 0.2?

Checkpoint 4: Put away the equipment and have your instructor check your work before leaving the lab.

Component	**Explanations**	**Points**	**Mark**
Worksheet	• If you finish all checkpoints, you will get 4 points.	4	
Group	• All students must be engaged in the lab activity. • All students must work, discuss, and share their information in the lab. • Interaction with group members and TA is mandatory. • All students must obtain answers to the questions that are the same as the other group members.	3	
Individual	• All appropriate data must be collected. • Data must be well organized and neatly displayed, including graphs. • The results of calculations must be presented with appropriate units. • Related physics concepts must be stated correctly.	3	

Please note that:

- Not properly cleaning the worktable or not putting away equipment that was taken out will result in a 1-point deduction from the "group" component of all members' grades.
- As of Labatorial 2 onward, not bringing your labatorial manual (in which case, a separately printed worksheet will be provided) or pre-reading summary to the lab will result in a 1-point deduction from the "individual" component of your grade.
- Progressing as a group is critical to the success of the labatorial, and so being more than 15 minutes late will result in a 1-point deduction from the "individual" component of your grade. Being more than 20 minutes late means you cannot perform the labatorial and you will receive a 0.