Labatorial 6: The Period of a Pendulum

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Pre-Reading:

- *Physics for Scientists and Engineers* by Serway and Jewett (10^{th} ed.), Section:
    - 1.3 – Dimensional Analysis
- http://spiff.rit.edu/classes/phys369/workshops/w2c/slope_uncert.html

Equipment: Photogate timer, strings of varying length, cylinders of varying mass

Learning Goals:

- Understanding what affects the period of a simple pendulum
- Understanding the applications of dimensional analysis
- Understanding how to estimate the error in the slope of a graph

Activity 1: A Dimensional Analysis Derivation (40-50 min.)

Problem: Suppose you are an up-and-coming physicist, and you are trying to understand pendulum motion. While you have a solid understanding of fundamental Newtonian physics concepts, you have not yet formally studied pendulums, and so you do not yet know what equations can describe its motion. In particular, you would like to know what affects the period of a pendulum. However, determined to figure things out, you hear about the technique of dimensional analysis while reading online and decide to give it a try. Then, to verify your result, you run some experiments.

Question 1:

a. A simple pendulum accelerates the way it does because of gravity. Therefore, the formula for the period of a pendulum must depend on g. What are the two other most likely variables that might influence the period of oscillation of a simple pendulum? (Feel free to verify these with the TA if you are unsure.) Write their units in terms of meters, kilograms, and seconds.

b. Based on your physical intuition, predict why and how you think each of these quantities might have an effect on the period.

c. If the period T, for example, is equal to a power formula depending on some variables, then the total units on both sides of the equation should match. But since we do not know what the power of each variable in the formula should be, we can temporarily make them variables as well. For example, if T is a function of the variables x, y, and z, then we can assume it to be of the form $T = x^a y^b z^c$. Once you have replaced your three variables with their respective units, group together the common units by combining the terms with a common base.

d. By matching the powers of fundamental dimensions (m, kg, and s) on each side of the mystery period equation, extract a system of linear equations based on the exponents and solve it for the correct powers. (Hint: The dimensions of T are $m^0 kg^0 s^1$.)

e. Suppose that the previously derived equation indeed gives us the correct dimensional relationships between the period and the relevant variables. What is one important piece of information that this method does not tell us? Write out the predicted formula for T, including the unknown piece of information. (Hint: This analysis only gives us proportionality relationships.)

Checkpoint 1: Before moving on to the next part, have your instructor check the results you obtained so far.

Activity 2: Verifying the Prediction (40-50 min.)

How to Use a Stand-Alone Photogate Timer:

When using a photogate for this experiment, one should set up the pendulum such that when the bob is hanging still, the photogate beam strikes its center as closely as possible. There is one primary function that will be of concern to this labatorial.

Because our goal is to measure the period of a pendulum, then the photogate timer should be set to PEND mode and the 1 ms position. Release the pendulum bob from a small angle and let the pendulum oscillate for about 1 minute. Then by pressing the RESET button, the period of the pendulum will be displayed on the timer. However, note that the timer should not be reset while the beam is striking the bob, as an erroneous reading will be given. As the pendulum swings, the RESET button can be pressed several times in order to obtain multiple period measurements, allowing you to compute an average period. The uncertainty of this average will then be

$$\Delta\overline{T} = \frac{\Delta T}{\sqrt{n}}, \qquad (1)$$

where ΔT is the uncertainty of the photogate readings and n is the number of measurements.

Question 2:

a. In light of the theoretical results we derived in the previous part, we would like to confirm the relationships between the period of the pendulum and the relevant variables. Using the photogate for the period measurements, devise the appropriate procedures and then make any necessary measurements or calculations, recording all data neatly in tables with uncertainties. Use the graph paper at the end of the labatorial for your plots. (Note that you have **two** relationships to check, and thus you should briefly describe two separate procedures.)

b. Using the plot(s) drawn for Question 2a, graphically determine the missing piece of information from our dimensional analysis. (Do not consider uncertainties for now.)

c. Consider again your predictions from Question 1b. Do they match with the relationships you just verified? Discussing with your teammates, explain why or why not.

Checkpoint 2: Before moving on to the next part, have your instructor check the results you obtained so far.

Activity 3: Analyzing the Results (30-40 min.)

<u>Review of How to Estimate the Uncertainty in the Slope of a Graph</u>:

When trying to draw a linear fit to data, it is not always clear what the correct slope should be due to the variations in the measurements. In particular, the points may not form a perfectly straight line, and each data point may possess some uncertainty in its x and/or y components. We can visually represent this x and/or y uncertainty by drawing error bars, which are bars centered on each data point extending outward in both directions by the magnitude of the uncertainty (left and right for an x error, and up and down for a y error).

Since error bars give you an idea of where the true point should lie, you can estimate bounds for how big or small the slope can reasonably be, which will allow you to calculate an uncertainty estimate for the slope. To do this, you must (not necessarily in the following order):

1. Draw the "best" line through all the points, taking into account the error bars. Measure the slope of this line.
2. Draw the "min" line -- the one with as small a slope as you think reasonable (taking into account error bars), while still doing a fair job of representing all the data. Measure the slope of this line.
3. Draw the "max" line -- the one with as large a slope as you think reasonable (taking into account error bars), while still doing a fair job of representing all the data. Measure the slope of this line.

Then the uncertainty in the slope a can be calculated as:

$$\Delta a = \frac{a_{max} - a_{min}}{2} \qquad (2)$$

More often than not, the slope of your graph will be equal to your quantity of interest multiplied or divided by a constant. In this case, the uncertainty of the quantity of interest will scale with the constant. In general, for a quantity of interest Q:

$$\text{If } Q = cX, \text{ then } \Delta Q = c\Delta X. \qquad (3)$$

Question 3:

a. Using the uncertainties of your period measurements, add vertical error bars to your graph.

b. Draw the “min” and “max” lines for your data, and calculate their slopes.

c. Calculate the uncertainty of the slope calculated in Question 2b using Equation 2, and then the uncertainty of the unknown quantity using Equation 3.

d. Ask the instructor for the exact period relationship and compare your experimental result to the theory. Does the equation you derived agree with the theoretical one within uncertainty? That is, is the expected result located within the range allowed by the experimental uncertainty?

Checkpoint 3: Have your instructor check your work before leaving the lab.

Component	Explanations	Points	Mark
Worksheet	• If you finish all checkpoints, you will get 4 points.	4	
Group	• All students must be engaged in the lab activity. • All students must work, discuss, and share their information in the lab. • Interaction with group members and TA is mandatory. • All students must obtain answers to the questions that are the same as the other group members.	3	
Individual	• All appropriate data must be collected. • Data must be well organized and neatly displayed, including graphs. • The results of calculations must be presented with appropriate units. • Related physics concepts must be stated correctly.	3	

Please note that:

- Not properly cleaning the worktable or not putting away equipment that was taken out will result in a 1-point deduction from the “group” component of all members’ grades.
- As of Labatorial 2 onward, not bringing your labatorial manual (in which case, a separately printed worksheet will be provided) or pre-reading summary to the lab will result in a 1-point deduction from the “individual” component of your grade.
- Progressing as a group is critical to the success of the labatorial, and so being more than 15 minutes late will result in a 1-point deduction from the “individual” component of your grade. Being more than 20 minutes late means you cannot perform the labatorial and you will receive a 0.