

Rock Paper Scissors Lizard

In this exercise students explore the concept of an evolutionarily stable strategy by examining a scenario where none exists; as seen in the Side-blotched lizards (*Uta stansburiana*) of the American south west (see included article and video links).

The mating system of these lizards is non-transitive. Each of the three morphs outcompetes one morph and is outcompeted by another as with rock paper scissors. In order to provide an example of non-transitivity that does not rely on an arbitrary system, specially constructed dice are used.

From Wikipedia: en.wikipedia.org/wiki/Nontransitive_dice

Consider the following set of dice.

- Die A has sides 2, 2, 4, 4, 9, 9.
- Die B has sides 1, 1, 6, 6, 8, 8.
- Die C has sides 3, 3, 5, 5, 7, 7.

If this game is played with a transitive set of dice, it is either fair or biased in favor of the first player, because the first player can always find a die that will not be beaten by any other dice more than half the time. If it is played with the set of dice described above, however, the game is biased in favor of the second player, because the second player can always find a die that will beat the first player's die with probability $5/9$. The following tables show all possible outcomes for all 3 pairs of dice.

Player 1 chooses die A
Player 2 chooses die C

C \ A	2	4	9
3	C	A	A
5	C	C	A
7	C	C	A

Player 1 chooses die B
Player 2 chooses die A

A \ B	1	6	8
2	A	B	B
4	A	B	B
9	A	A	A

Player 1 chooses die C
Player 2 chooses die B

B \ C	3	5	7
1	C	C	C
6	B	B	C
8	B	B	B

Three-dice set with minimal alterations to standard dice

The following nontransitive dice have only a few differences compared to 1 through 6 standard dice:

- as with standard dice, the total number of pips is always 21
- as with standard dice, the sides only carry pip numbers between 1 and 6
- faces with the same number of pips occur a maximum of twice per die
- only two sides on each die have numbers different from standard dice:
 - A: 1, 1, 3, 5, 5, 6

- B: 2, 3, 3, 4, 4, 5
- C: 1, 2, 2, 4, 6, 6

Each student is invited to make their own die, following one of three patterns, and invited to roll against the other two patterns while keeping track of the outcomes. Finally, the groups, one per pattern, pool their data and vote on which die is the best, i.e. which one in their estimation wins the most.

The die can be made from paper boxes (template included) or using the minimal alteration set described above, regular die can be modified with permanent marker (these roll more effectively).

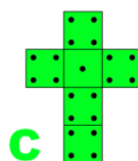
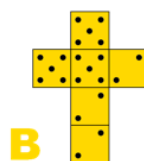
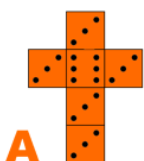
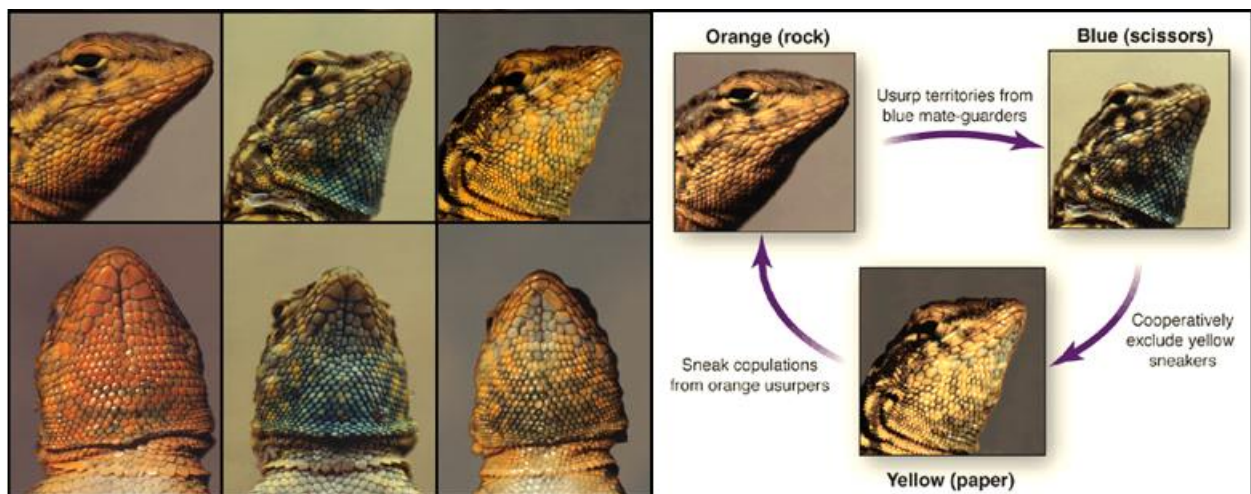
It is recommended that each pattern is shared with only one group. The information regarding the total value of all the faces for each pattern and that the average expected value for each die is equal can be shared.

The win/lose rates of the die are only subtly biased in one die's favour so a large sample is required. The included matrix above can be used to explain the expected outcome. If the class results are far off the expected values it can be used to comment on the nature of stochasticity in biology.

The nature of the dice is discussed and the principles which underlie it are applied to the case of the side blotch lizards. The side blotch lizards are an excellent example of frequency dependent selection.

See included paper: An experimental test of frequency-dependent selection on male mating strategy in the field (Bleay et al).

The existence of such dice often seems counterintuitive at first introduction, the novelty of encountering them aids in making the exercise memorable.



Cube Pattern

Cut on solid lines - Fold on dashed lines

